

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3425

SOME CALCULATIONS OF THE LATERAL RESPONSE OF TWO
AIRPLANES TO ATMOSPHERIC TURBULENCE WITH RELATION TO THE
LATERAL SNAKING PROBLEM

By John D. Bird

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SUMMARY

Calculations are made of the lateral response to representative time histories of atmospheric turbulence for two airplanes having widely different dynamic properties, and explanations for their differences in behavior are given.

The results of the calculations indicate that, under the proper conditions, atmospheric turbulence can initiate and maintain a lateral hunting oscillation of an airplane, and that this oscillation can be fairly regular in both amplitude and frequency. This effect is more pronounced for lightly damped airplanes. It is felt that this phenomenon may be the cause for some of the cases of airplane snaking that have not been explained by other considerations.

INTRODUCTION

Several recent high-speed airplanes exhibit a tendency to develop and maintain lateral hunting oscillations of roughly constant amplitude, which are generally referred to as snaking oscillations. These oscillations are of essentially the same period as the normal Dutch roll oscillation of the airplane and usually have amplitudes of the order of ±1° yaw. They are considered to be a nuisance to the pilot in that they cause loss of confidence in the airplane's response to control and make the airplane less satisfactory as a gun platform.

Adequate explanations have been offered for this behavior in specific cases; however, there are still numerous occurrences for which no satisfactory explanation has been advanced. Some of the explanations for this motion are associated with nonlinear aerodynamic characteristics which result in different rates of damping for the large and

 $^{^{}m l}$ Supersedes the recently declassified NACA RM L50F26a, 1950.

small amplitude—ranges of motion. An example of this nature could arise from a poor fairing at the juncture of the tail surfaces. Other causes of snaking can be associated with small amounts of slack in the rudder control system, or the effects of fuel sloshing.

Some unpublished experiments conducted in the Langley stability tunnel have suggested the possibility that motions, similar to those described as snaking, may result from the turbulence which exists in the atmosphere. The present paper, therefore, constitutes a preliminary investigation of this possibility. Calculations are made of the lateral response to representative time histories of atmospheric turbulence for two airplanes, having widely different dynamic properties, and explanations for their differences in behavior are given. Calculations and experimental records of the response of a model, which has freedom only in yaw, to the roughness in the stability-tunnel air stream are given for illustrative purposes.

SYMBOLS

The stability system of axes is used in the present analysis. This axis system has its origin at the center of gravity. The Z-axis is in the plane of symmetry and perpendicular to the relative wind, the X-axis is in the plane of symmetry and perpendicular to the Z-axis, and the Y-axis is perpendicular to the plane of symmetry. The positive directions of the stability axes and of angular displacements of the airplane are shown in figure 1. The coefficients of forces and moments employed in this paper are in standard NACA form.

The coefficients and symbols are defined as follows:

$\mathtt{C}_{\mathbf{L}}$	lift coefficient, Lift/qS	
$C^{\mathbf{X}}$	lateral-force coefficient, Y/qS	··- <u>-</u>
Cl	rolling-moment coefficient, L/qSb	· - <u>-</u>
C_n	yawing-moment coefficient, N/qSb	-
$c^{\lambda \beta} = \frac{9\beta}{9c^{\lambda}}$	· Production of the control of the	<u></u>
$C_{l\beta} = \frac{\partial C_l}{\partial \beta}$		_ _

$$C_{n\beta} = \frac{\partial \beta}{\partial C_n}$$

$$CI^{b} = \frac{9\frac{5A}{bp}}{9C^{I}}$$

$$c^{\mathbf{n}^{\mathbf{b}}} = \frac{9\frac{5\Lambda}{\mathbf{b}p}}{9c^{\mathbf{u}}}$$

$$C^{IL} = \frac{9 \frac{Lp}{Lp}}{9 C^{I}}$$

$$C^{nL} = \frac{9\frac{sh}{Lp}}{9C^{u}}$$

 K_{X} dimensionless radius of gyration about X-axis, k_{X}/b

K_Z dimensionless radius of gyration about Z-axis, k_Z/b

 K_{XZ} dimensionless product-of-inertia factor, k_{XZ}/b^2

k_X radius of gyration about X-axis

k_{7.} radius of gyration about Z-axis

 \mathbf{k}_{XZ} product-of-inertia factor

 μ_{b} relative density factor, $\frac{W}{goSb}$

 $\mathbf{I}_{\mathbf{Z}}$ moment of inertia about Z-axis

p angle of bank of airplane, radians unless otherwise noted

w azimuth angle of airplane, radians unless otherwise noted

 β angle of sideslip of airplane, radians unless otherwise noted, $\tan^{-1}\frac{V}{V}$

σ angle of air stream with respect to initial flight-path direction of airplane, radians unless otherwise noted

 $\sigma_{_{\hbox{\scriptsize O}}}$ amplitude of oscillation in air-stream direction

 ψ_{O} amplitude of oscillation in model heading

 $\frac{1000}{V} \frac{\psi_0}{\sigma_0}$ relative amplification, V in feet per second

f frequency, cycles per unit time

t time

D_b differential operator, d/ds_b

sh dimensionless time, tV/b

s_{b1} a particular time

$$D_b^n = \frac{d^n}{ds_b^n}$$

η inclination of principal longitudinal axis of inertia with respect to flight path; positive when the principal axis is above flight path at nose

W weight

S wing area

b wing span

A aspect-ratio

g acceleration due to gravity

X longitudinal force along X-axis

Y lateral force along Y-axis

Z normal force along Z-axis, Lift = - Z

L rolling moment about X-axis

N yawing moment about Z-axis

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pb/2V	wing-tip helix angle, radians
rb/2V	yawing-velocity parameter, radian measure
p	rolling angular velocity about X-axis, radian measure
r	yawing angular velocity about Z-axis, radian measure
v	linear velocity of airplane along Y-axis
v gust	lateral gust velocity with respect to undisturbed position of airplane
ν	free-stream velocity
ď	dynamic pressure, $\frac{\rho}{2} V^2$
ρ	mass density of air
P	period of free lateral oscillation
^T 1/2	time for free lateral oscillation to damp to one-half amplitude

CALCULATION METHODS

General methods are given in references 1 and 2 for calculating the lateral response of airplanes to gusts. Reference 1 indicates that the response of an airplane to an arbitrary gust structure may be obtained by superposition of solutions for unit gusts, which, in the limiting case of a continuous disturbance function, involves the evaluation of Duhamel's integral. It is also pointed out in reference 1 that a rigorous analysis of gust effects requires consideration of penetration time and of the aerodynamic lag in building up the lift on the surfaces. Examination of some of the penetration effects indicated that their magnitudes were small compared with the effects of sideslip. The rolling component of the measured turbulence considered for this paper was, of course, unknown; and, thus, all gust disturbances were necessarily considered to be in a single plane. For the purpose of the calculations of this paper, the turbulence was assumed to contribute nothing more than an effective change in sideslip of the airplane. Thus, in a side gust

of velocity Vo, the angular and linear disturbances are $\sigma \frac{dC_n}{d\sigma}$, $\sigma \frac{dC_l}{d\sigma}$

and $\sigma \frac{dC_Y}{d\sigma}$ where the derivatives are numerically equal to C_{n_β} , C_{l_β} , and C_{Y_β} , respectively. In this analysis $\sigma \frac{dC_Y}{d\sigma}$ was assumed to be zero.

The lateral response of two airplanes to representative time histories of atmospheric turbulence was calculated by obtaining the motion of the airplanes following the application of unit yawing- and rolling-moment coefficients and then by evaluating Duhamel's integral with this unit solution as the response variable and the record of the lateral fluctuation in air-stream direction as the forcing function.

Now Duhamel's integral may be written

$$\psi sbl = \int_{0}^{sbl} \psi_{\sigma} \left(sbl - sb \right) \frac{d\sigma}{dsb} dsb$$

where $\psi_{\sigma}(s_{b_1} - s_b)$ is the lateral response of the airplane to a unit σ . This equation may be broken into two integrals:

$$\psi_{\mathbf{s}_{b_1}} = \int_{0}^{\mathbf{s}_{b_1}} \psi_{\mathbf{N}} \Big(\mathbf{s}_{b_1} - \mathbf{s}_{b} \Big) \Big(\frac{\mathrm{d} c_{\mathbf{n}}}{\mathrm{d} \sigma} \Big) \frac{\mathrm{d} \sigma}{\mathrm{d} \mathbf{s}_{b}} \ \mathrm{d} \mathbf{s}_{b} + \int_{0}^{\mathbf{s}_{b_1}} \psi_{\mathbf{L}} \Big(\mathbf{s}_{b_1} - \mathbf{s}_{b} \Big) \Big(\frac{\mathrm{d} c_{\mathbf{N}}}{\mathrm{d} \sigma} \Big) \frac{\mathrm{d} \sigma}{\mathrm{d} \mathbf{s}_{b}} \ \mathrm{d} \mathbf{s}_{b}$$

where $\psi_N\left(s_{b_1}-s_b\right)$ and $\psi_L\left(s_{b_1}-s_b\right)$ are the lateral responses of the airplane to unit yawing- and rolling-moment coefficients, respectively, and $dC_n/d\sigma$ and $dC_1/d\sigma$ are numerically equal to C_{n_β} and $C_{2\beta}$, respectively. The last equation, of course, may be expressed as a single integral:

$$\psi \, \mathbf{s}_{b_{\underline{1}}} \, = \, \int_{O}^{\mathbf{s}_{b_{\underline{1}}}} \, \left[\overline{\frac{d}{d} \mathbf{c}_{\underline{n}}} \, \psi_{\, \underline{N}} \left(\mathbf{s}_{b_{\underline{1}}} \, - \, \mathbf{s}_{\underline{b}} \right) \, + \, \frac{d \, \mathbf{c}_{\, \underline{1}}}{d \, \sigma} \, \psi_{\, \underline{L}} \left(\mathbf{s}_{b_{\underline{1}}} \, - \, \mathbf{s}_{\underline{b}} \right) \right] \, \frac{d \, \sigma}{d \, \mathbf{s}_{\, \underline{b}}} \, d \, \mathbf{s}_{\underline{b}}$$

The solutions to the lateral equations of motion following the application of unit yawing- and rolling-moment coefficients were obtained by use of an automatic digital computing machine and the procedures of reference 2. Duhamel's integral was evaluated by a numerical integration in a manner similar to that of reference 3 to obtain the motion of the airplane in response to the turbulence. This calculation was also carried out on an automatic computing machine.

A brief résumé of the methods used for calculating the lateral frequency-response characteristics of the airplanes considered is given in the appendix along with the equations of motion from which the response of the airplanes to unit disturbances was calculated.

RESULTS AND DISCUSSION

Calculated Airplane Motions in Turbulent Air

The mass and aerodynamic characteristics of two airplanes having widely different operating conditions and dynamic characteristics are given in table I. Airplane A is a low-speed, low-altitude airplane; and airplane B is a high-speed, high-altitude research airplane, which is known to exhibit small continuous lateral oscillations under certain flight conditions.

Figures 2 and 3 show the results of the calculation of the lateral response of airplanes A and B to two known distributions of atmospheric turbulence. The turbulence shown in figure 3 was measured by means of a sensitive recording pitot-static tube mounted on an airplane which traversed a region of turbulent air and so recorded the fluctuations in forward speed through the region. The turbulence shown in figure 2 was measured by means of an accelerometer mounted at the center of gravity of an airplane which traversed a region of turbulent air. Both of these records of turbulence were considered to be fluctuations in sidewise velocity for these calculations. Reference 4 gives information on the measurement of atmospheric turbulence and justification for the assumption that the turbulence is isotropic.

The gust distributions shown in the figures were assumed to exist in like fashion along the flight paths of both airplanes. The high-speed airplane, of course, encounters gusts with a greater frequency than the low-speed airplane. The gust velocities are assumed for these calculations to be the same for all altitudes. These results are thus of a qualitative nature.

The distance traversed by the airplanes is used as an abscissa in the plots given, and the azimuth angles of the airplanes are chosen to indicate the oscillation performed. The azimuth angle should be roughly proportional to the apparent lateral movement of the horizon.

The motions of the two airplanes in response to the two regions of atmospheric turbulence are markedly different (figs. 2 and 3). Airplane A shows a response which might logically be termed by the pilot as rough air; that is, the airplane responds in almost direct proportion to the local gustiness and subsides to little or no motion as the gustiness subsides. The response of most airplanes in the past seems to have been of this nature. For example, see the small amplitude motions measured in flight during the investigation reported in reference 5. Airplane B shows a response which builds up to a fairly steady lateral oscillation, which is almost independent of the local turbulence. This motion is very similar in character to motions which have been termed snaking.

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The periods of the lateral oscillations, shown in figures 2 and 3, are close to the period of the classical free-lateral oscillations of the airplanes (table I). This is more nearly the case for airplane B than airplane A.

Figure 4 shows the motions of both airplanes compared with the angular motion of the air with respect to the undisturbed attitude of the airplane for the case of the turbulence determined from airspeed fluctuations. It should be noted that the turbulence record corresponds to different amplitudes of σ for the two airplanes because of differences in forward speed. It is easy to note a closer approximation in the case of airplane A to a one-to-one correspondence of air motion to airplane motion than for the case of airplane B, where there is little apparent relation between the air and airplane motions. It should be mentioned here that the long period change in heading shown for airplane A in figure 3 was modified to some degree in the preparation of figure 4 in order to have the air and airplane motions oscillate about the same mean and so make for an easier comparison of the two motions.

A comparison of a part of the lateral motion of airplane B, as calculated from the atmospheric turbulence (fig. 3), with the snaking of this airplane recorded during a flight test is shown in figure 5. This comparison merely confirms the statement made previously that lateral motions arising from turbulence in the air can be similar in nature to flight measurements of snaking motions. The flight conditions for the two motions given are not identical, and the atmospheric turbulence existing during the flight test is unknown. The indications are, however, that turbulence having about one-third the magnitude of that employed for figure 3 would be required to maintain a hunting motion of airplane B comparable with the snaking motion that it exhibited in flight. It should be pointed out that the disturbances do not have to be of the type generally referred to as sharp-edged gusts as shown in figure 2, but may be of a more gentle nature, as shown in figure 3.

Free Motions of Model in Wind Tunnel

As an example of the response of a free body to turbulence in the air stream, figure 6 gives the experimental and calculated hunting motion of a model mounted with freedom only in yaw in the air stream of the Langley stability-tunnel test section. The measured time history of the air-stream azimuth angle from which the model motion was calculated is also shown. The calculation procedure was much the same as has been given previously for airplanes A and B. The time history of airstream direction was obtained by use of a recording electronic pitot and, although the percentage error in the magnitude of the air-stream angles may be fairly large, the nature of the fluctuations should be accurate. The experimental model motion was not recorded at the same time as the

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air-stream azimuth angle. This fact does not change the nature of the result, however, because the air-stream turbulence was of a similar nature for a number of different recordings.

The tunnel model was free only in yaw, was mounted on flexure plates in order to minimize friction, and consisted of only a fuselage and vertical tail. The mass and dimensional characteristics of the model oscillating system are given in table II.

It can be seen that the experimental and calculated motions of figure 6 are of a similar nature, indicating that the air-stream turbulence is a significant factor in the hunting motion experienced by the model.

Frequency-Response Characteristics

The response of airplanes A and B, and of the tunnel model mounted with freedom in yaw, to sinusoidal forcing functions of various frequencies is given in figure 7. The forcing functions are in the form of changes in heading of the approaching air stream. The results are given in terms of the amplitude of motion in radians induced by a sinusoidal lateral gust distribution having an amplitude of 1000 feet per second and in terms of the phase lag of the motion behind the forcing function.

An examination of these curves indicates the source of the difference in response of airplanes A and B to atmospheric turbulence. Airplane B has a frequency response not unlike the characteristics of an electronic band-pass filter which excludes those harmonics of the applied frequency which are very much different from the resonant frequency. A good measure of the selectivity of response of these airplanes is the ratio of the amplification at the natural frequency to the amplification at very low frequencies. Airplane A responds to a greater degree than airplane B to those frequencies that are different from the resonant frequency; thus, the tendency for the one-to-one correspondence of air direction to airplane azimuth angle shown for airplane A in figure 4. In general, the sharper the frequency-response curve the more nearly the response to atmospheric turbulence approaches a sinusoidal motion.

Any effect which reduces the rate of free damping of an airplane should tend to increase the peak of the frequency-response curve and make the phase-angle shift at the natural frequency more abrupt. Airplane B, of course, has a low rate of damping for the condition investigated herein (table I).

The rate of free damping can be affected to a marked degree by a change in one or more of the aerodynamic stability derivatives. This

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fact would make calculations, based on estimated derivatives, of a questionable nature unless some form of check is available. The stability derivatives used for the calculations of this paper are believed to be reasonably accurate because the experimental and calculated rates of $T_{1/2}$ and P of the lateral oscillation compare well (table I).

The frequency-response curve of the oscillating model mounted in the air stream of the langley stability tunnel is between the curves of airplanes A and B with regard to selectivity but is large compared with both as regards over-all response (fig. 7).

CONCLUDING REMARKS

The results of the calculations of the lateral response of two airplanes to atmospheric turbulence indicated that, under the proper conditions, atmospheric turbulence can initiate and maintain a lateral hunting oscillation of an airplane, and that this oscillation can be fairly regular in both amplitude and frequency. This effect is more pronounced for lightly damped airplanes. It is suggested that this phenomenon may be the cause for some of the cases of airplane snaking that have not been explained by other considerations.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 27, 1950.

APPENDIX

CALCULATION OF LATERAL FREQUENCY RESPONSE

The lateral frequency response of an airplane to an imposed sinusoidal variation of wind direction may be calculated by solving the standard lateral equations of motion of an airplane (with the proper forcing terms added) for the steady-state motion. For a unit sinusoidal variation in air-stream direction the amplitude of this motion becomes the amplification factor or the amount that the air-stream fluctuation is magnified.

Within the limits of the approximations discussed, the lateral equations of motion of an airplane experiencing a sinusoidal variation in air-stream direction are

$$\begin{split} & \left(2\mu_{b}K_{X}^{\ 2}D_{b}^{\ 2} - \frac{1}{2}\ C_{l_{p}}D_{b}\right)\phi \ + \left(2\mu_{b}K_{XZ}D_{b}^{\ 2} - \frac{1}{2}\ C_{l_{r}}D_{b}\right)\psi \ - \ C_{l_{\beta}}\beta \ = \ \sigma_{o}C_{l_{\beta}}\ \sin \ 2\pi \ \frac{fb}{V}\ s_{b} \\ & \left(2\mu_{b}K_{XZ}D_{b}^{\ 2} - \frac{1}{2}\ C_{n_{p}}D_{b}\right)\phi \ + \ \left(2\mu_{b}K_{Z}^{\ 2}D_{b}^{\ 2} - \frac{1}{2}\ C_{n_{r}}D_{b}\right)\psi \ - \ C_{n_{\beta}}\beta \ = \ \sigma_{o}C_{n_{\beta}}\ \sin \ 2\pi \ \frac{fb}{V}\ s_{b} \\ & - \ C_{L}\phi \ + \ 2\mu_{b}D_{b}\psi \ + \ \left(2\mu_{b}D_{b} \ - \ C_{Y_{\beta}}\right)\beta \ = \ 0 \end{split}$$

where the terms $C_{l\beta}$ and $C_{n\beta}$ on the right side of the first two equations are considered to be the equivalents of $\frac{dC_l}{d\sigma}$ and $\frac{dC_n}{d\sigma}$.

The variation of side force with rolling and yawing velocity, a term associated with the glide-path angle, and the side-force forcing function are omitted in these equations. Calculations showed these factors to be of little importance. Replacing the two right-hand terms of these equations by unity gives the equations from which the unit solutions were obtained for use in the calculations of response to arbitrary turbulence.

Solving the equations for the steady-state motion gives for the azimuth angle $\,\psi\,$

$$\psi = \sqrt{a^2 + b^2} \sin \left[\left(\frac{2\pi fb}{V} \right) s_b - \epsilon \right]$$

where ϵ is the angle of lag of the motion behind the disturbance, and for $\sigma_0 = 1.0$, the term $\sqrt{a^2 + b^2}$ is the amplification factor for the given imposed frequency or

$$\frac{\psi_0}{\sigma_0} = \sqrt{a^2 + b^2}$$

Now the amplitude of the yawing motion for a given amplitude of lateral gust velocity is

$$\psi_{O} = \left(\frac{\psi_{O}}{\sigma_{O}}\right) \left(\frac{v_{gust}}{V}\right)$$

This term is called the relative amplification when 1000 is substituted for $v_{\hbox{gust}}$ and V is given in feet per second. Now

$$a = \frac{k_{e}C_{e} - k_{u}C_{u}}{C_{e}^{2} + C_{u}^{2}}$$

and

$$b = \frac{k_e C_u + k_u C_e}{C_u^2 + C_e^2}$$

where

$$k_e = m_2 f_s^2 + m_0$$

$$k_u = m_3 f_s^3 + m_1 f_s$$

$$C_e = Bf_s^4 - Df_s^2$$

$$C_u = -Af_s^5 + Cf_s^3 - Ef_s$$

and

$$f_{s} = \frac{2\pi fb}{V}$$

$$m_0 = 0$$

$$\begin{split} \mathbf{m}_{1} &= \left(\frac{1}{2} \, \mathbf{C}_{l_{\mathbf{p}}} \mathbf{C}_{\mathbf{Y}_{\beta}} \mathbf{C}_{\mathbf{n}_{\beta}} \, - \frac{1}{2} \, \mathbf{C}_{\mathbf{n}_{\mathbf{p}}} \mathbf{C}_{\mathbf{Y}_{\beta}} \mathbf{C}_{l_{\beta}}\right) \sigma_{o} \\ \\ \mathbf{m}_{2} &= \left[\left(\mu_{b} \mathbf{C}_{l_{\mathbf{p}}} + 2\mu_{b} K_{\mathbf{X}}^{2} \mathbf{C}_{\mathbf{Y}_{\beta}}\right) \mathbf{C}_{\mathbf{n}_{\beta}} \, - \left(\mu_{b} \mathbf{C}_{\mathbf{n}_{\mathbf{p}}} + 2\mu_{b} K_{\mathbf{X}\mathbf{Z}} \mathbf{C}_{\mathbf{Y}_{\beta}}\right) \mathbf{C}_{l_{\mathbf{\beta}}}\right] \sigma_{o} \\ \\ \mathbf{m}_{3} &= \left(-\frac{1}{2} \mu_{b}^{2} \mathbf{E}_{\mathbf{X}\mathbf{X}}^{2} \mathbf{C}_{\mathbf{n}_{\beta}} + \frac{1}{2} \mu_{b}^{2} \mathbf{E}_{\mathbf{X}\mathbf{Z}} \mathbf{C}_{l_{\mathbf{\beta}}}\right) \sigma_{o} \end{split}$$

The expressions for the coefficients of the lateral-stability equation A, B, C, D, and E are given in reference 6.

The lag angle is given by

$$\epsilon = \cos^{-1} \frac{a}{\sqrt{a^2 + b^2}} = \sin^{-1} \frac{b}{\sqrt{a^2 + b^2}}$$

Substitution of various values of the imposed frequency in the previously given expressions gives the frequency-response curve.

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TABLE I

MASS AND AERODYNAMIC CHARACTERISTICS OF AIRPLANES

	Airplane A Airplane B
μъ	16.8 106.3
K _X ²	0.0061 0.0051
K_Z^2	0.0264 0.0409
κ_{XZ}	0.0058 -0.0006
b, feet	33.6 28 8,700 11,050 250 130 0.551 0.343
c_{l_p}	0.551 0.343 -0.280 -0.474
c_{n_p}	-0.085 0
c_{l_r}	0.090 0.224
c_{n_r}	-0.270 -0.170
C _{1β}	-0.049 -0.101
$c_{n_{\beta}}$	0.097 0.217
$c_{Y_{\beta}}$	-0.665 -0.878
V, feet per second	257 746 7,500 30,000 3.7 1.48 3.5 1.80 2.5 6.56 3.0 6.50

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TABLE II

MASS AND AERODYNAMIC CHARACTERISTICS OF MODEL MOUNTED

WITH FREEDOM IN YAW IN AIR STREAM OF THE

LANGLEY STABILITY TUNNEL

I _Z , f	cot-po	und-1	вес	ond	2	•	•	•					•	•	•	•	•	•	•	•	•		•		•		0.3
	und per																										
*b, f	eet .					•	•		•					•	•	•	•	•	•	•	•	•	•		•	•	3
V, fe	et per	seco	ond	•	٠	٠	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	186
*S, s	quare 1	feet					٠																				2.25
$\mathtt{c_{n_{B}}}$	per rad	lian	•	• •	•	•	•	٠	•	•	•	٠	•	•	•	•	•	•	•	٠	•	•	•	•	•	•	0.063
$\mathtt{c_{n_r}}$.			. ,								•		•		٠	•	•		٠.			•		•	•	•	-0.13
P (ca	.ļculate	∍d),	sec	con	ds.			•		•		•	•	•	•	•	•	•		•		•		•	•	•	0.8
^T 1/2	(calcul	Lated	1),	se	COI	nds	3	•	٠	٠	٠	•	•	٠	•	•	•	•	•	•	•	•	•	•	•	•	4.4
	*The s	ymbo	ls	ъ	a	nd		s	а	re	g	iv	en	a	s	đi	me	ns	10	ns	u	po	n	wh	ic	h	
aero	dynamic																				•	_					A

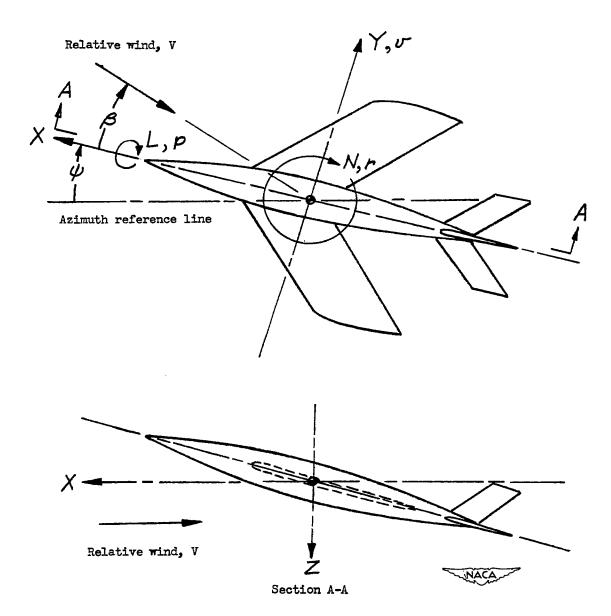


Figure 1.- Stability system of axes. Positive values of forces, moments, velocities, and angles are indicated by arrows.

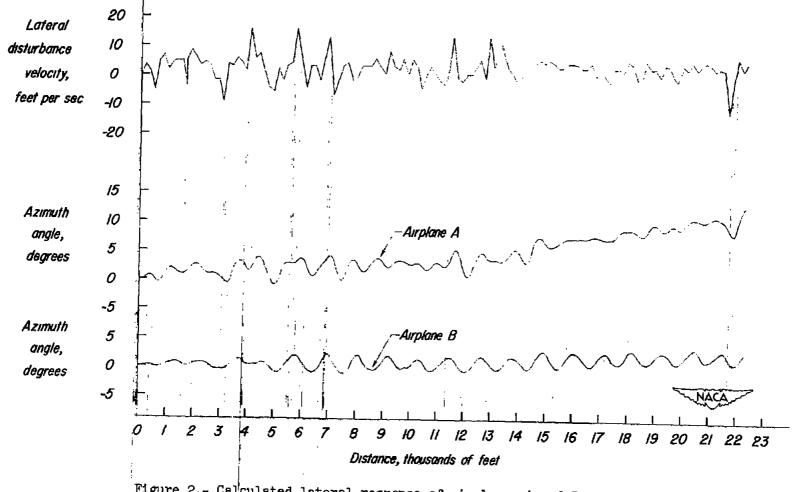


Figure 2.- Calculated lateral response of airplanes A and B to atmospheric turbulence. Turbulence obtained from accelerometer records.

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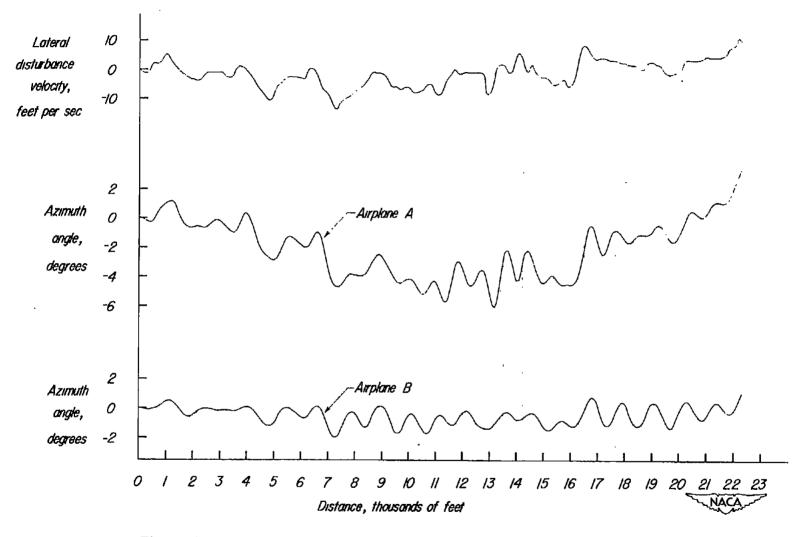


Figure 3.- Calculated lateral response of airplanes A and B to atmospheric turbulence. Turbulence obtained from airspeed fluctuation records.

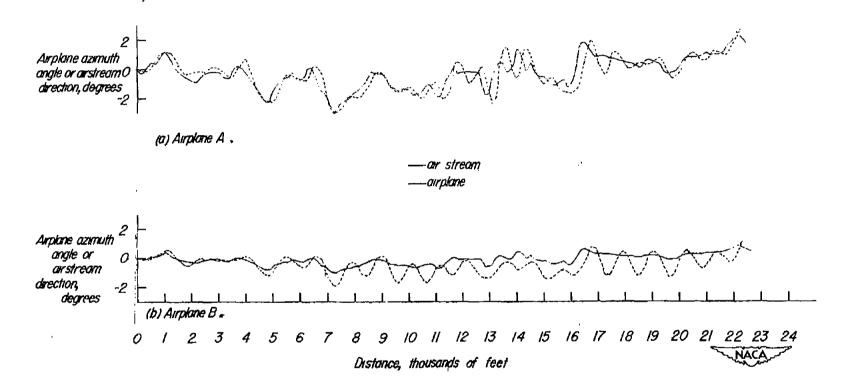
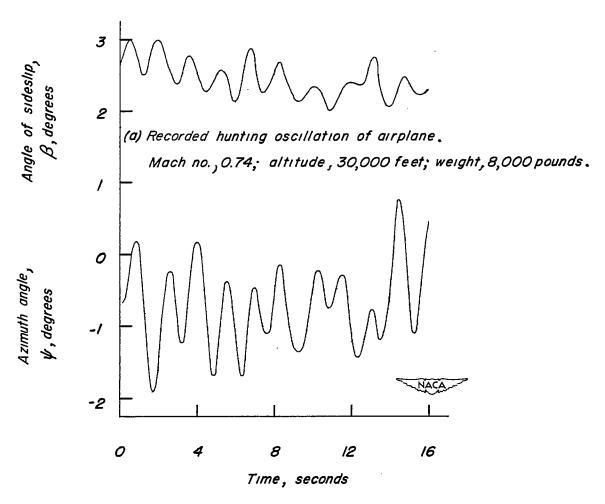


Figure 4.- Comparison of lateral motions of airplanes A and B with fluctuation in air-stream direction. Turbulence distribution of figure 3.

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(b) Calculated response of airplane to atmospheric turbulence (from figure 3).

Mach no. , O. 74; altitude, 30,000 feet; weight, II,000 pounds.

Figure 5.- Comparison of calculated hunting motion of airplane B in response to atmospheric turbulence with measured hunting motion of airplane B obtained from a flight test.

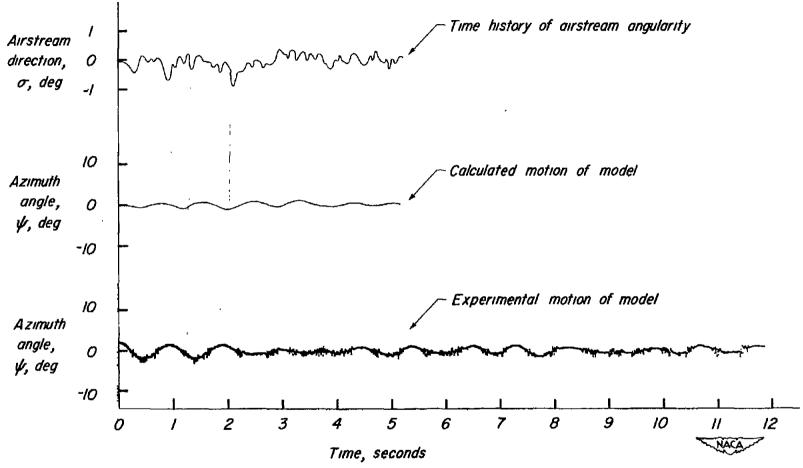


Figure 6.- Comparison of hunting motion of model mounted with freedom in yaw in stability-tunnel air stream with calculated hunting motion determined from a representative record of air-stream turbulence. $q = \frac{1}{4}0$ pounds per square foot.

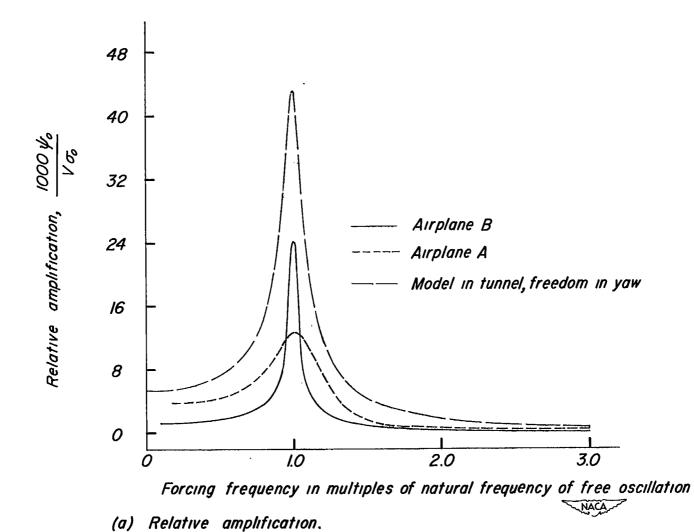
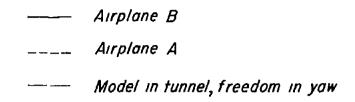
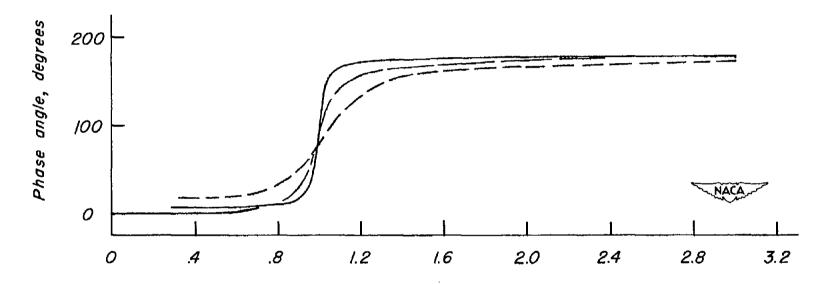


Figure 7.- Lateral-frequency-response characteristics of airplanes A and B and model mounted with one degree of freedom in stability-

tunnel air stream.





Ratio of impressed frequency to natural frequency of lateral oscillation

(b) Phase lag of motion behind impressed moment.

Figure 7.- Concluded.

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